Matlab Exercises

Lecture 5 – Bayesian tracking with Kalman Filters and Condensation

1) **Kalman Filter implementation**

Write a Matlab function that implements the two steps of Kalman Filter (prediction+correction):

A. Prediction: Given a Gaussian motion model with covariance matrix Λ_w, and linear matrix A, compute the predicted (prior) state s^- and covariance matrix S^-
Inputs: Matrix A, covariance Λ_w, old posterior mean s_{t-1} and covariance S_{t-1}
Outputs: new Prior mean and covariances, (s^-,S^-)

B. Correction: Given a Gaussian measurement model with covariance matrix Λ_v, and linear matrix C, compute the corrected (posterior) state s_t and covariance matrix S_t
Inputs: Matrix C, covariance Λ_v, prior mean s^- and covariance S^-
Outputs: new posterior mean and covariances, (s_t,S_t)

2) **Kalman Filter example**

With the previously implemented functions, now test the Bayesian tracker (Kalman) for the following case:

Suppose to have a random point moving on a 2D plane, with a random WNA motion:

$s_t = As_{t-1} + w_t$

with Λ_w = diag(0,0,1,1) (the noise is only in acceleration, so it goes into the velocity equations, not in the position!)

Suppose the initial state is also not known, and has a prior probability distribution $P_0(\mathbf{s})$ = Gauss(**0**,10), that is: the initial state is all zero (2D pose+velocity) both with uncertainty σ^2=10.

The measurement z is a position measurement: z = Cs+v, where C = [I 0] is a 4x2 matrix that takes only the upper part of **s** (i.e. the pose), plus a 2D measurement uncertainty v=Gauss(**0**,1).

With the given model, do the following parallel things:

A. Simulate the random process:

- Give a random initial state s_0 according to P_0 (use the Matlab function *randn()* to generate Gaussian random numbers)
- At time t, apply the motion model (1) by generating a random acceleration w_t, and

updating the real state s_t.
- At time t, simulate also the measurement $z_t = Cs_t+v_t$ by generating random 2D Gaussian number v.

B. Apply the Kalman Filter:

- At time 0, use only the correction function, with the prior knowledge : s_0- = [0,0], S_0-=diag(10,10,10,10)

- At time t, use both prediction+correction functions developed in the previous exercise.

C. Compare the real state with the Kalman estimation (plot a graph of the state components in time: x(t), y(t) and x,y velocity)

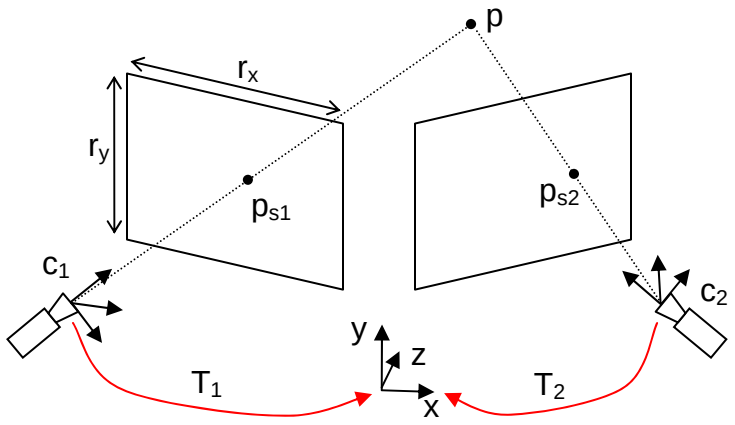
- At each time, compute the difference between the real state (simulated in A) and the estimated posterior state obtained in B (the pose only is sufficient)

- Plot the results on a 2D graph: the real trajectory $\mathbf{s}_0,\mathbf{s}_1,\ldots$ and the estimated one (posterior), again only the pose.

3) **Extended Kalman Filter: track a flying ball (DLR system)**

Suppose to have two cameras (a stereo system), looking a ball thrown across the room.

The setup is the one described in Exercise 3-Lecture 4, as below indicated



The ball **p**=(x,y,z) describes a parabolic trajectory during the flight, and its motion model can be described by a constant gravity acceleration towards the bottom (-**g**) + a small random component w (e.g. air resistance in different points of the trajectory).
This motion (described in Lecture 4 - Slide 10) gives a probabilistic state model:
$P(\mathbf{s}_t|\mathbf{s}_{t-1}) = \text{Gauss}\,(A\,\mathbf{s}_{t-1}+C,\, B\boldsymbol{\Lambda}_w B^T)$.

with

$$A = \begin{bmatrix} I & I\Delta t \\ 0 & I \end{bmatrix} \qquad B = \begin{bmatrix} I\Delta t^2 \\ I\Delta t \end{bmatrix} \qquad C = \begin{bmatrix} -\mathbf{g}I\Delta t^2 \\ -\mathbf{g}I\Delta t \end{bmatrix}$$

($A\,\mathbf{s}_{t-1}$+C) is the *prediction* of $\mathbf{s}_t$
$\mathbf{g}$ = [0 981 0] is the gravity acceleration (y direction)
Λ_w = diag(0,0,0,1,1,1) is the covariance of motion noise (acceleration noise)
Δt = 0.1 is the time sampling interval (10 frames/sec).

The state **s** is a (3+3)-vector (position+velocity), and positions are measured in [*mm*].
The measurement **z** (solution of the other exercise) is the collection of two positions located on the two camera images:
$\mathbf{z} = (\mathbf{p}_{s1}, \mathbf{p}_{s2})$, which are 4 image coordinates (x_1,y_1,x_2,y_2).

The measurement model, for a given hypothesis **p**, gives an expected measurement

$$p_{s1,\exp}(p_{c1}) = \left(\frac{x_{c1}}{z_{c1}} f + \frac{r_x}{2}, \frac{y_{c1}}{z_{c1}} f + \frac{r_y}{2} \right) \qquad p_{s2,\exp}(p_{c2}) = \left(\frac{x_{c2}}{z_{c2}} f + \frac{r_x}{2}, \frac{y_{c2}}{z_{c2}} f + \frac{r_y}{2} \right)$$

$$p_{c1} = (x_{c1}, y_{c1}, z_{c1}) = T_1 p \qquad p_{c2} = (x_{c2}, y_{c2}, z_{c2}) = T_2 p$$

where $\mathbf{p}_{c1}$ and $\mathbf{p}_{c2}$ are the coordinates of p in the two cameras (extrinsic transformations T_1, T_2), and $\mathbf{p}_{s1,exp}$, $\mathbf{p}_{s2,exp}$ are the projections on the screens (intrinsic transformation: f, r_x, r_y).

The parameters for this example are the following ones:

T_1: only translation to the left t_x = -100mm
T_2: only translation to the right t_x = +100mm
f = 1000, r_x = 640 pixels, r_y = 480 pixels
Λ_v = covariance of measurement noise = I (1 pixel uncertainty)

The probabilistic measurement model is (nonlinear $\mathbf{z}_{exp}$+Gaussian), therefore an Extended Kalman Filter can be used for Bayesian tracking.

A. Compute the Jacobian matrix J = $\frac{\partial \mathbf{z}_{\exp}}{\partial \mathbf{s}}$ at given hypothesis **s**, (write a Matlab function returning J, with input **s**)

B. Implement the Extended Kalman Filter (equations in Lecture 5-Slide 19).
NOTE: the motion model is already linear, so the Jacobian is just A.

C. Do a simulated experiment (real vs. estimated state), where the ball is thrown from the ground:

Real initial state $\mathbf{p}_0$ = [0,0,0] with initial velocity $\mathbf{v}_0$ = [0, 10, 10] (forward z, up y).
Initial state hypothesis: $\mathbf{p}_0$ = [0,0,0], $\mathbf{v}_0$ = [0,0,0] (no knowledge).
Perturbation of acceleration during the flight = Gaussian random **w**, with covariance 1.

Run the EKF, and report the results as for the Kalman filter (trajectories).

4) **Particle Filters implementation**

Implement a basic Particle Filter, by defining the 3 functions :

A. Re-sample: given a N-particles set (s^i,π^i), take a new particle set obtained by sampling N times between (1,…,N) (with evtl. repetitions) with probabilities $(\pi^1,\ldots,\pi^N)$.

B. Move: for every re-sampled particle, apply a motion model $s_t = g(s_{t-1},w_t)$, for example a WNA motion with given acceleration covariance. For this purpose, generate a random acceleration for every particle (hypothesis).

C. Re-weight: give new weights $\pi^i = P(z|s^i)$ to the moved particles, by using a Likelihood function P(z|s) given by the user.

5) **Particle Filters example**

Apply the Particle Filters implementation to the same example used for Kalman Filter; here the Likelihood is Gaussian: $P(z|s) = Gauss(C\mathbf{s},\Lambda_v)$.